Renewable Energy 65 (2014) 245–249

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Characteristics of turbine spacing in a wind farm using an optimal design process

Eunkuk Son [a,*], Seungmin Lee [a], Byeongho Hwang [a], Soogab Lee [b]

[a] Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Republic of Korea
[b] Center for Environmental Noise and Vibration Research, Engineering Research Institute, Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Republic of Korea

ARTICLE INFO

Article history:
Received 6 February 2013
Accepted 17 September 2013
Available online 4 October 2013

Keywords:
Wake model
Wind farm efficiency
Optimal turbine spacing
Genetic algorithm

ABSTRACT

The characteristics of turbine spacing for optimal wind farm efficiency were investigated using combined numerical models. The effects of wakes from upstream turbines were predicted by a model capable of determining velocity distributions on a rotor plane, based on Ainslie's approach. The performance results of a wind farm showed good agreement with measurements. The blade element momentum theory, in combination with a dynamic wake model, was applied. Wake model used the results of aerodynamic analysis as input properties. The optimal distance between wind turbines was predicted using a genetic algorithm to maximize efficiency in a wind farm. The results showed that the spacing between the first and the second turbines had the importance to the entire farm's efficiency.

1. Introduction

In spite of the global economic depression, the wind energy market has been continuously growing by over 20% annually [1]. One of the main reasons for this promotion of economic growth in wind energy is the establishment of large wind farms. Moreover, since the number of large offshore wind farm has been rapidly increasing since the 2000s, this growth trend in wind energy is anticipated to last for several decades.

There are some known technical barriers to wind farm development, such as effect of wakes generated by upstream turbines. Wind speed is decreased in wakes due to momentum loss caused by an upstream turbine; the reduced velocity causes power loss for the downstream turbine. The level of turbulence intensity also continuously increases due to the upstream turbine wakes, which in turn decreases the lifetime of the turbine system.

Wake models have been developed in order to predict flow properties that have decisive effects on system life and performance of a wind farm. There are simple approach models such as Jensen's linear expansion model as well as sophisticated models such as Ainslie's eddy viscosity model (EVM) and Crespo's UPMWAKE model. Although UPMWAKE model is regarded as an elaborate computation method with great capability for velocity fields and particularly turbulence intensity in wakes, it is unsuitable for application to an optimal design stage [2].

Ainslie's EVM, by contrast, cannot predict variations in turbulence intensity in the wake region; however, it has a much lower computation cost and provides accurate results of velocity distributions. Therefore, this model is appropriate for wind farm layout design.

In previous research related to wind farm layout optimization, a linear expansion model that overestimated wake effects was widely used for its simplicity. Since the model uses approximations to superpose wake effects, it underestimates wind speed in the wind farm and consequently results in extremely large spacing between the turbines [3,4]. Therefore, a model based on Ainslie's approach has been used for the prediction of wake velocity in far wakes and BEMT in combination with dynamic wake model has been used to estimate turbine performance in the wake region. With these numerical cycles, characteristics of turbine spacing in wind farms have been investigated.

2. Numerical methods

2.1. Rotor aerodynamic analysis

Aerodynamic analysis tool based on blade element momentum theory in combination with dynamic wake model was used for the prediction of wind farm performance. A steady state solution can be estimated with general BEMT, which assumes a turbine rotor as several annular sections and uses one dimensional momentum equation. It is inaccurate when wind speed varies with height, as it

* Corresponding author. Tel.: +82 2 880 7384; fax: +82 2 876 4360.
E-mail address: sonddol@snu.ac.kr (E. Son).

0960-1481/$ – see front matter © 2013 Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.renene.2013.09.022

Nomenclature

BEMT	blade element momentum theory
EVM	eddy viscosity model
F	filter function
$G(r)$	Gaussian profile
GA	genetic algorithm
K_1	dimensionless constant of atmosphere parameter
K_M	eddy diffusivity of momentum
N	number of azimuthal rotor section
N_{wt}	number of wind turbine
$P_{i,j}(U)$	probability density function
$U_{\infty\psi}$	free stream velocity in rotor azimuthal angle
WT	wind turbine
b	wake width
f	object function
u	velocity component in longitudinal direction
v	velocity component in lateral direction
r	radial direction in rotor blade
u_∞	free stream velocity
u_c	centerline velocity
u_*	friction velocity
$u(z)$	velocity as a function of height
ψ_w	wind velocity component of Businger-Dyer relationships
ψ	azimuthal angle
κ	von Karman constant

depends on the assumption of equal wind speed at each annular section. Furthermore, the steady state solution of BEMT will no longer follow the trend for a large wind turbine, as the velocity difference is much larger. In order to improve capability of general BEMT method, dynamic wake model was applied. It determines the sectional angle of attack with the induced velocity of a blade in case of unsteady inflow conditions [5,6].

2.2. Wake model

The eddy viscosity model (EVM) introduced by Ainslie (1988) was used for the prediction of a velocity profile in wakes. The momentum equation approximated from turbulent boundary layer equation with eddy viscosity closure is as follows:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r} = \frac{\varepsilon}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad (1)$$

Eddy viscosity, ε, is modeled with wake width and velocity difference at centerline as length scale and velocity scale, respectively, and they are related as follows: [7]

$$\varepsilon = FK_1 b(u_\infty - u_c)+K_M \quad (2)$$

In order to improve on the original EVM which only shows the velocity deficit in longitudinal direction, boundary conditions at all azimuthal locations are differently applied.

$$U_{\infty_\psi} = \begin{bmatrix} u_{\infty_{\psi_1}} \\ u_{\infty_{\psi_2}} \\ \vdots \\ u_{\infty_{\psi_N}} \end{bmatrix}, u_{\infty_{\psi_i}} = u(z)_{\infty_{\psi=(2\pi/N)i}} G(r);\ i=0\sim N,\ 0\le\psi_i\le 2\pi \quad (3)$$

$G(r)$ describes the Gaussian profile, the ratio of wake velocity to free stream velocity, suggested by Ainslie [8].

Inflow velocity in terms of height is determined by some parameter such as Z_0 and ψ_w. These values affect the vertical distribution of the wind speed even in the same wind speed at the hub height. A typical value of roughness length in the sea state was used and ψ_w was determined by the Businger-Dyer relationships in accordance with the atmospheric stability. The rigorous derivation and expression can be found in Stull's work (1988) [8].

$$u(z) = \frac{u_*}{\kappa}\left(\ln\frac{z}{z_0}-\psi_w\right) \quad (4)$$

2.3. Optimization technique

An optimal design process was carried out using genetic algorithm (GA), known as one of the best approaches for solving a nonlinear system. The object function was defined by a cost model and performance properties. The cost model proposed by G. Mosetti (1994) was used, and the total power value was chosen from one of the performance properties [3]. The object function used in this work was defined as follows:

$$f = \frac{\mathrm{Cost}(N)}{\mathrm{Total\ Power}} \quad (5)$$

3. Results

3.1. Wind farm performance comparison

Comparison of numerical results with measurements is conducted for the simple validation of the present computation model. Reference measurements for wind velocity with height were from Schepers's ECN report (2003), and reference values for turbine performance were taken from R.J. Barthelmie et al. (2009) [9,10].

The velocity profiles with atmospheric stability were compared with measurements from the Vindeby wind farm. The turbine, Bonus 450 kW, has a 35 m rotor diameter and 38 m hub height. Wind speed in the validation cases is equal to 5 m/s with 6% of turbulence intensity. The validation cases were divided into three atmospheric states such as a stable, a neutral, and an unstable condition. Specification of the measurement site and detailed experimental conditions can be found in Schepers's report [9]. The normalized wake velocity with the measurement value is shown Fig. 1.

The distance between a turbine and a met mast was 9.6 diameters (D). The red line with the square symbol shows the distribution of free flow. The black line with a filled black square symbol shows the measurement value in the wake region; the velocity for far wakes is normalized by the free stream velocity at hub height. The results had good agreement, with less than 3% error for all atmospheric conditions. In particular, for stable atmospheric condition, the present results showed better performance compared with ECN's calculation, which is shown as a blue (in web version) dotted line.

The velocity below the hub height region was less accurate compared with that of above the hub height region; this difference was due to errors in the numerical inflow velocity in the vicinity of the ground.

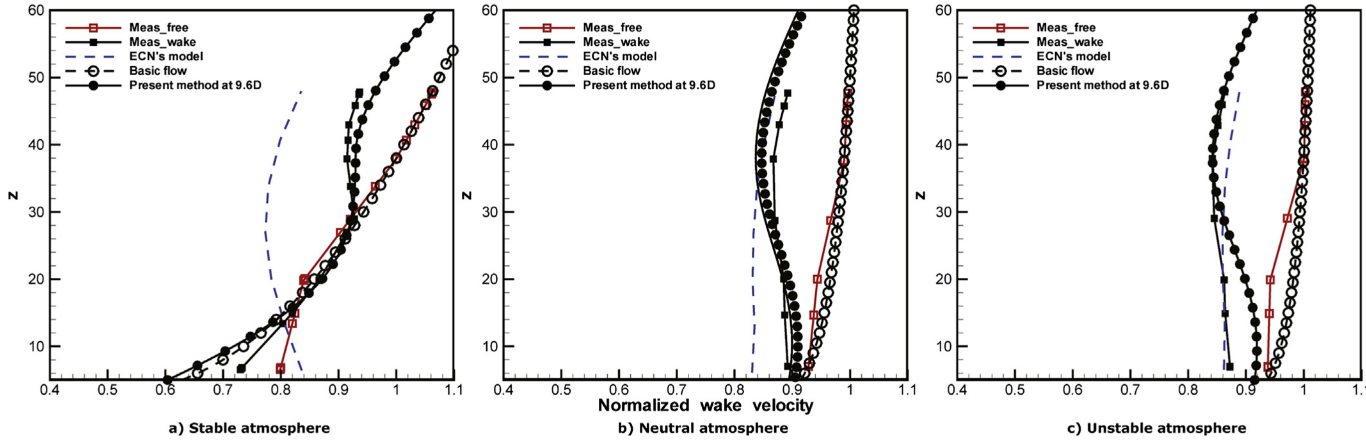

Fig. 1. Normalized wake velocity comparison with Vindeby wind farm measurement.

Fig. 2 shows the normalized power of the turbines in wakes. These results were compared to measurements from Horns Rev as well as other numerical results [10,11]. These values were obtained from the previously mentioned numerical approaches, the aerodynamic model and the wake model. The numerical values from the present method had good agreement with the measurement values, unlike in other models. The difference between the present methods and the others cannot exactly find out because the name of the models was intentionally hidden in the original work [9]. The better agreement of the present methods might be caused by the combined procedure capable of the accurate calculation for the velocity profiles less than 3% and azimuthal consideration for the aerodynamic loading calculations.

3.2. Turbine spacing optimization

Optimization of turbine spacing was conducted in order to maximize power output from a wind farm. Since the only primary wind direction and exact row cases were considered, wake effects in the lateral direction were neglected. This means that downstream turbines were only affected by upstream turbines in the same column. Considering the spacing in common wind farm, we had chosen six wind turbines in optimization procedure.

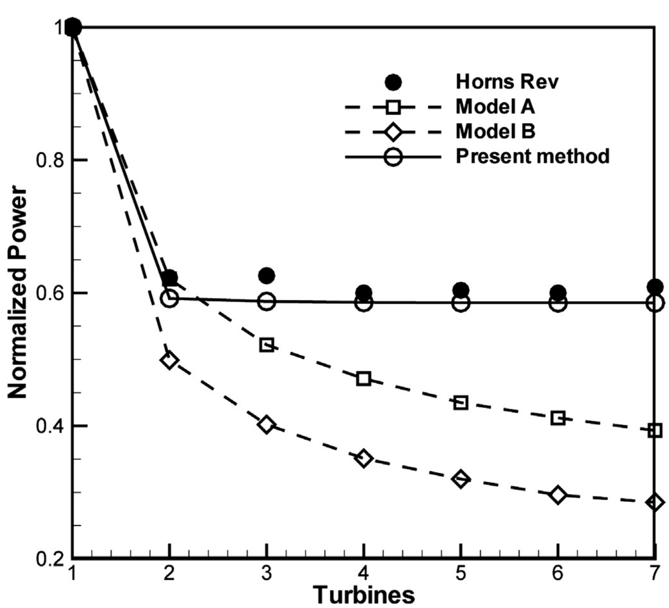

Fig. 2. Comparison of normalized power with Horns Rev's measurement (270°, Case 1 in Ref. [10]) and the other numerical models.

The object function in this work was equation (5) and the cost model, equation (6) proposed by G. Mosetti et al. (1994), was chosen. Although the cost model is simple, it has been widely used because its assumption that 1/3 of cost reduction can be obtained when a large number of turbines are installed is quite reasonable in the economical point of view [3].

$$\text{Cost} = N_{\text{wt}}\left(\frac{2}{3}+\frac{1}{3}e^{-0.00174N_{\text{wt}}^2}\right) \quad (6)$$

At first, the information on the local site, such as atmospheric conditions, is needed if an optimal design is conducted in all wind speed range. We only considered the average wind speed in the atmospheric properties. The wind farm simulation was performed considering 3 MW turbines at rated wind speed, 12.5 m/s, and the turbine spacing was regarded as the control variable. Each turbine spacing for optimization results was constrained to 4D $\leq$ Turbine spacing $\leq$ 15D.

The variation in the object function and the normalized total power at each wind speed are shown in Fig. 3. When the wind speed was above 14.0 m/s, the average wind speed in wakes was found to reach the rated wind speed. For this reason, the obtained power output was consistently in the range from 14.0 m/s to 18.0 m/s.

The optimized results including turbine spacing, power output, thrust coefficient, and power coefficient are listed in Table 1. The turbine spacing is represented as a number with a non-dimensional diameter index (*D*). The first turbine in normal working conditions had no wake effects, while the others had effects of velocity deficits, which caused power losses. When the distance between the first wind turbine (WT_1) and the second wind turbine (WT_2) became larger, the power output dropped dramatically in comparison with the normal state. However, when the distance became closer, the WT_2 had severely low efficiency in terms of energy extraction and the momentum of flow in wakes was not insufficient to produce power. As such, turbines should be spaced as closely as possible, while still leaving sufficient space so that the power output for the downstream turbines is guaranteed.

4. Discussion and conclusion

4.1. Object function characteristics

The variation of the object function in terms of changes of the turbine spacing was explored. Each line, expressed by $WT_{(n)} \sim WT_{(n+1)}$, displays the variation of the object function in

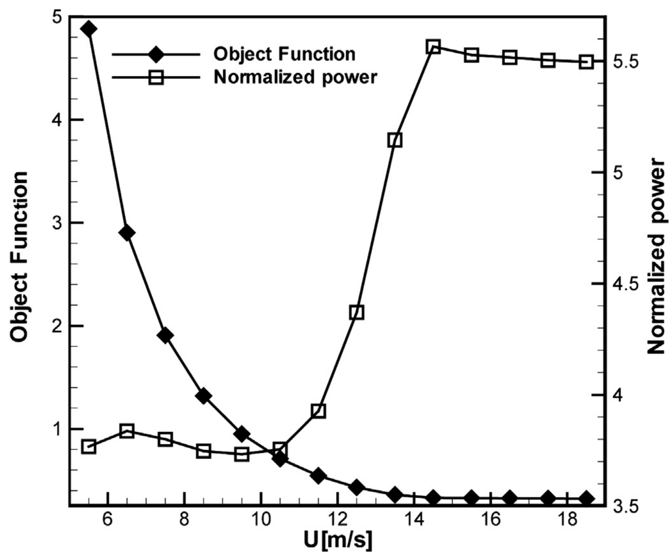

Fig. 3. Variation of the object function and the normalized power with the average wind speed.

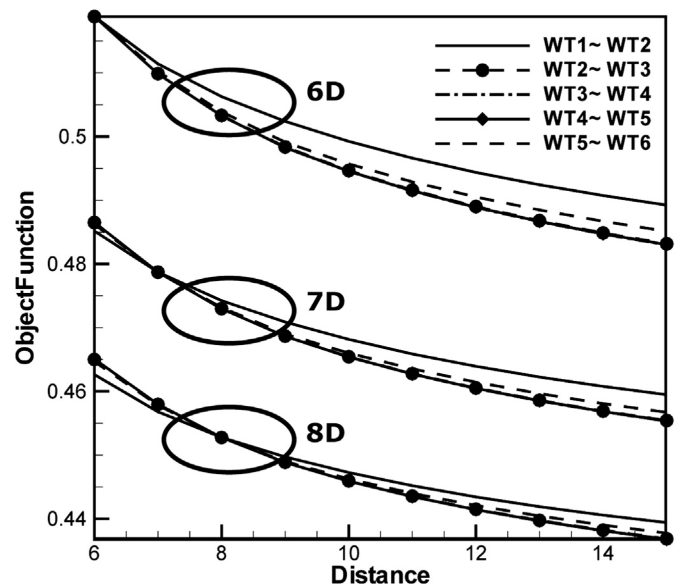

Fig. 4. Contribution of turbine spacing to the object function.

Fig. 4. The object function at each line can be obtained when the turbine spacing $WT_{(n)} \sim WT_{(n+1)}$ changes whereas the spacing among the other turbines has the same distance. For example, the black solid line, $WT_1 \sim WT_2$, at the top of the three groups show the object function variation with spacing changes from 6D to 15D whereas the spacing among other turbines keep the same distance, 6D. Similarly, the results of middle group are obtained by the constraint with 7D among other turbines and those of the bottom group are from the constraint with 8D.

Based on the results shown in Fig. 4, the spacing between WT_1 and WT_2 has a small contribution to the object function's change. The small contribution to object function means that there is a small improvement of a total power while the turbine spacing becomes longer. A variation of the object function of the black line, $WT_1 \sim WT_2$, is −0.02960 during 6D to 15D. However, the variation of the black line with a dot, $WT_4 \sim WT_5$, has −0.03581. It shows that the way of increasing the spacing between WT_4 and WT_5 is a better choice in terms of the improvement of the total power because the object function decreases when the total power increases. Consequently, increasing the spacing between WT_1 and WT_2 is inefficient, in comparison to increasing the spacing between WT_4 and WT_5. From the results, the optimal spacing results can be confirmed at Table 1. The spacing between the first and the second turbine is smallest and the other spacing is almost equal.

4.2. Conclusion

An optimal design process with implementing numerical methods to determine the turbine spacing that results in maximum efficiency in a wind farm is presented. The optimal distribution of the turbine spacing having 0.147537 of the object function in Table 1 shows better performance compared to the commercially used turbine spacing having 0.157483, the same distance among the turbines. An aerodynamic analysis tool using the blade element momentum theory in combination with the dynamic wake model was used for the prediction of blade sectional performance. Sectional inflow velocity at the rotor plane in turbine wakes was obtained using a model based on turbulent boundary layer equation with the eddy viscosity closure proposed by Ainslie (1988) [7]. The wake model was able to calculate for the case of wind shear from the ground to the top of the rotor, unlike the original EVM. The velocity profile in far wakes has less than 3% of errors compared with the measurement value at three different atmospheric conditions. This velocity profiles were used as the input profile for a downstream wind turbine. These cycles of aerodynamic analysis were repeatedly conducted until the wake reaches the final turbine. Characteristics of wind turbine spacing have been investigated using the genetic algorithm. Since the first wind turbine has maximum thrust coefficient because it exposes in free stream condition, the performance of the second turbine is significantly reduced. However, the increase of the spacing between the first and the second turbine is an inefficient way in terms of the total power of a wind farm as shown Fig. 4. In order to determine the optimal spacing, it can guarantee the maximum efficiency of a wind farm.

Table 1
Property of the optimal design in wind farm.

No. of turbine	Power (kW)	C_t	C_p
1	3119.57	0.5654	0.4184
2 (7D)	2191.45	0.7487	0.4534
3 (11D)	2098.15	0.7607	0.4584
4 (11D)	2086.28	0.7614	0.4585
5 (11D)	2085.50	0.7615	0.4585
6 (10D)	2052.15	0.7619	0.4576
Total power (kW)	13633.10	Object function	0.431198
Cost	5.8786		

Acknowledgment

This work was supported by the Human Resources Development program (No. 20124030200030) and the New & Renewable Energy Technology Development Program (No. 20123010020130) of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Trade, Industry and Energy.

References

[1] Global wind report – annual market update 2011. GWEC; 2012.
[2] Crespo A, Manuel F, Moreno D, Fraga E, Hernandez J. Numerical analysis of wind turbine wakes. In: Bergeles G, Chadjivassiliadis J, editors. Proceedings of the Delphi workshop on wind energy applications, Delphi, Greece 1985. p. 15–25.

[3] Mosetti G, Poloni C, Diviacco B. Optimization of wind turbine positioning in large windfarms by means of a genetic algorithm. J Wind Eng Ind Aerodyn 1994;51:105–16.
[4] Grady SA, Hussaini MY, Abdullah MM. Placement of wind turbines using genetic algorithms. Renew Energy 2005:259–70.
[5] Hansen MO. Aerodynamics of wind turbine. UK: Earthscan; 2008.
[6] Øye S. Dynamic stall, simulated as a time lag of separation. In: McAnulty KF, editor. Proceedings of the 4th IEASymposium on the aerodynamics of wind turbines, ETSU-N-118, Harwell Laboratory, Harwell, UK 1991.
[7] Ainslie JF. Calculating the flow field in the wake of wind turbines. J Wind Eng Ind Aerodyn 1988;21:213–24.
[8] Stull Roland B. An introduction to boundary layer meteorology. Kluwer Academic Publishers; 1988.
[9] Schepers JG. ENDOW: validation and improvement of ECN's wake model. ECN-C; March 2003. p. 03–034.
[10] Barthelmie RJ, Hansen K, Frandsen ST, Rathmann O, Schepers JG, Schlez W, et al. Modelling and measuring flow and wind turbine wakes in large wind farms offshore. Wind Energy 2009;12:431–44.
[11] Son Eunkuk, Lee Jaejoon, Kim Taehyung, Lee Seunghoon, Lee Soogab. Wind turbine aerodynamic analysis in non-axisymmetric wake flow. In: Proceedings of 1st Aisa-Pacific forum on renewable energy 2011.